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A sufficient condition for blowup solutions of nonlinear heat equations ☆

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Abstract

The author discusses the initial-boundary value problem $(u_i)_t = \Delta u_i + f_i(u_1, \dots, u_m)$ with $u_i|_{\partial\Omega} = 0$ and $u_i(x, 0) = \phi_i(x)$, $i = 1, \dots, m$, in a bounded domain $\Omega \subset R^n$. Under suitable assumptions on f_i, he proves that, if $\phi_i \geqslant (1 + \varepsilon_0)\psi_i$ in $D_i \subset \Omega$, for some small $\varepsilon_0 > 0$, then the solutions blow up in a finite time, where ψ_i is a positive solution of $\Delta\psi_i + f_i(\psi_1, \dots, \psi_m) \geqslant 0$, with $\psi_i|_{\partial D_i} = 0$ for $i = 1, \dots, m$. If $m = 1$, the initial value can be negative in a subset of Ω.

Keywords: Parabolic equation; Dirichlet problem; Blowup solutions

Consider the following nonlinear heat equations:

$$\begin{cases} (u_i)_t = \Delta u_i + f_i(u_1, \dots, u_m), & t > 0,\ x \in \Omega, \\ u_i(t, x) = 0, & t > 0,\ x \in \partial\Omega, \\ u_i(0, x) = \phi_i(x), & x \in \Omega, \end{cases} \tag{1}$$

for $i = 1, \dots, m$, where $\Omega \subset R^n$ is a bounded domain with smooth boundary $\partial\Omega$. For convenience, we denote $\mathbf{u} = (u_1, \dots, u_m)$.

It is well known that, for some small initial values, the solutions exist globally, while for some large initial values the solutions blow up in finite time if $f_i(\mathbf{u})$ increases superlinearly (see [3,7,10,12]). A natural question is what kind of initial values produce solutions of (1) that blow up in finite time.

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If $m = 1$ and $f(u) = u^p$ with $p > 1$, Levine [11] proved that solutions of (1) must blow up in finite time, as long as $\phi(x)$ is large enough in the sense that its "energy"

$$E(\phi) = \frac{1}{2}\|\nabla\phi\|_2^2 - \frac{1}{p+1}\|\phi\|_{p+1}^{p+1}$$

is negative. Tan [13] gave a sharp condition on $\phi(x)$ such that the solution of (1) blows up in finite time when p is a critical Sobolev exponent. Lacey [10] showed that for general $f(u)$, the solution blows up if $\phi(x)$ is greater than a nonminimal steady state and $f', f'' \geqslant 0$, $\int^\infty s/f(s)\,ds < \infty$. This is also a sharp result.

If $m = 2$ and $\mathbf{f}(\mathbf{u}) = ((1-u_2)g(u_1), (1-u_2)g(u_1))$ with $g(u_1) = e^{u_1}$, Bebernes and Lacey [2] have shown that solutions of (1) blow up in finite time. Later, in [3], they extended their results to more general $g(u_1)$. If $\mathbf{f}(\mathbf{u}) = (u_1^{m_1}u_2^{n_1}, u_1^{m_2}u_2^{n_2})$, Chen [5] proved that if $m_1 \leqslant 1$, $n_1 \leqslant 1$, and $m_2n_1 \leqslant (1-m_1)(1-n_2)$ then all nonnegative solutions are global, while if $m_1 > 1$ or $n_1 > 1$ or $m_2n_1 > (1-m_1)(1-n_2)$ then both global existence and finite time blowup coexist. Later, Wang [14] used a different, more simple method to improve Chen's results. In particular, if either

$$m_1 > 1, \quad n_1 > 0, \quad m_2 = 0, \quad n_2 = 1, \quad \lambda < 1, \quad m_1 \leqslant 1 + n_1(1-\lambda)/\lambda$$

or

$$n_2 > 1, \quad m_2 > 0, \quad n_1 = 0, \quad m_1 = 1, \quad \lambda < 1, \quad n_2 \leqslant 1 + m_2(1-\lambda)/\lambda$$

then for any nonnegative initial data the solution blows up in finite time, where λ is the first eigenvalue of $-\Delta$ in Ω with homogeneous Dirichlet boundary condition.

Gang and Sleeman [7] gave several sufficient conditions to obtain the blowup property in the one-dimensional parabolic system with $m = 2$,

$$\frac{\partial u_i}{\partial t} = \alpha_i \frac{\partial^2 u_i}{\partial x^2} + f_i(u_1, u_2), \quad -a < x < a,\ \alpha_i > 0,\ i = 1, 2.$$

Pao [12] studied more general systems with two equations. He assumed that f_1 and f_2 are quasimonotone nondecreasing,

$$f_i(x, u_1, u_2) \geqslant \sigma_i u_1^{m_i} u_2^{n_i} - c_i u_i, \quad i = 1, 2,$$

for $u_i > 0$, and there exist positive constants $\mu \geqslant 1$ and $\nu \geqslant 1$ such that

$$\gamma = \min\bigl\{\mu(m_1 - 1) + \nu n_1, \mu m_2 + \nu(n_2 - 1)\bigr\} > 0.$$

Then for any $(\phi_1, \phi_2) \geqslant (\omega^\mu, \omega^\nu)$, the solution of (1) must blow up in finite time, where ω satisfies

$$\Delta\omega + \sigma(\omega)^{1+\gamma} - c_0\omega \geqslant 0 \quad \text{in } \Omega, \qquad \omega = 0 \quad \text{on } \partial\Omega,$$

with $c_0 = \max\{(c_1/\mu, c_2/\nu\}$ and $\sigma = \min\{(\sigma_1/\mu, \sigma_2/\nu\}$.

Chen and Derrick [4] considered the system

$$(u_i)_t = \Delta u_i + f_i(u_1, \ldots, u_m), \quad t > 0,\ x \in \Omega,$$

with $u_i|_{\partial\Omega} = 0$, $u_i(x, 0) = \phi_i(x)$ and $m \geqslant 1$. Under suitable assumptions on the nonlinear terms f_i, they proved that, if $0 \leqslant \phi_i \leqslant \lambda\psi_i$ with $\lambda < 1$, then the solutions are global, while if $\phi_i \geqslant \lambda\psi_i$ with $\lambda > 1$, then the solutions blow up in a finite time, where ψ_i are positive solutions of $\Delta\psi_i + f_i(\psi_1, \ldots, \psi_m) = 0$, with $\psi_i|_{\partial\Omega} = 0$. Later, Chen [6] extended similar results to a parabolic equation with a gradient term.

In this paper, we generalize the blowup result of [4] by removing the condition $\psi_i|_{\partial\Omega} = 0$. If $m = 1$ we also allow the initial value to be negative in a subset of Ω. We assume that

(i) f_i is locally Lipschitz continuous, $f_i(\mathbf{u}) \geqslant 0$ if $\mathbf{u} \geqslant \mathbf{0}$ and $f_i(\mathbf{u})/u_i > f_i(\mathbf{v})/v_i$ whenever $\mathbf{u} > \mathbf{v}$, $\mathbf{u} > \mathbf{0}$ and $v_i > 0$ for $i = 1, \ldots, m$.

(ii) There exists an index k, $1 \leqslant k \leqslant m$, such that $f_k(\mathbf{u})/u_k^\sigma \geqslant c_0 > 0$ for some $\sigma > 1$, $u_k > 0$ and all $u_i > c_1 > 0$ with $i \neq k$.

(iii) $D_i \subset \Omega$ is an open domain in R^n with a closed, simple and smooth boundary ($\partial D_i \cap \partial\Omega$ may be nonempty) for $i = 1, \ldots, m$ and there exists a small ball B_r in Ω such that $\bar{B}_r \subset \bigcap_{i=1}^m D_i$. ψ_i is defined in Ω and is a positive solution of $\Delta\psi_i + f_i(\boldsymbol{\psi}) \geqslant 0$ in D_i, with $\psi_i|_{\partial D_i} = 0$ for $i = 1, \ldots, m$. Furthermore, $\psi_i \leqslant 0$ on $D_j - D_i$ if $D_j - D_i \neq \emptyset$ and $i \neq j$.

(iv) $\phi_i(x) \in C^{1+\alpha}(\bar{\Omega})$ with $\phi_i|_{\partial\Omega} = 0$ for $\alpha > 0$ and $i = 1, \ldots, m$.

Theorem 1. *Under the hypotheses* (i)–(iv), *if $\phi_i(x) \geqslant (1 + \varepsilon_0)\psi_i(x)$ in D_i for some small $\varepsilon_0 > 0$ and $\phi_i(x) > 0$ in Ω for $i = 1, \ldots, m$, then the solution $\mathbf{u}$ of* (1) *must blow up in finite time.*

Proof. From standard parabolic PDE theory, there exists a unique solution

$$\mathbf{u}(x,t) \in C^{1,0}\big(\bar{\Omega} \times [0, T^*)\big) \cap C^{2,1}\big(\bar{\Omega} \times (0, T^*)\big)$$

to (1), where T^* is the maximal time such that $\mathbf{u}(x,t)$ exists and is bounded (see [1] or [8]). By the maximum principle, $u_i \geqslant 0$. Let

$$g_i^n(t) = \int_{D_i} \frac{\psi_i^{n+2}(x)}{u_i^n(x,t)}\, dx \tag{2}$$

for $t \in [0, T^*)$ and any real number $n > 0$. The following method was first used in [4] where the integrals are defined on the whole domain Ω. Here each integral is defined on a different subdomain D_i.

If ∂D_i is located inside of Ω, then (2) is well defined because $u_i(x,t) > 0$ on $\bar{D}_i$. Since u_i is continuous in Ω, there exists $\bar{t} > 0$ such that $u_i(x,t) \geqslant \psi_i(x)$ in D_i for $0 < t \leqslant \bar{t}$ and $1 \leqslant i \leqslant m$. If there is a point $x_0 \in \partial\Omega \cap \partial D_i$, then

$$\frac{1}{1+\varepsilon_0} \geqslant \lim_{x \to x_0} \frac{\psi_i(x)}{\phi_i(x)} = \lim_{x \to x_0} \frac{\psi_i(x) - \psi_i(x_0)}{\phi_i(x) - \phi_i(x_0)} = \frac{\partial\psi_i(x_0)/\partial n}{\partial\phi_i(x_0)/\partial n}, \tag{3}$$

where $x \in \Omega$ and is located on the normal line that passes through x_0. Since $\partial u_i(x_0,t)/\partial n < 0$ by the strong maximum principle and $\partial u_i(x,t)/\partial n$ is continuous on $\bar{\Omega}$, we have

$$\lim_{x \to x_0} \frac{\psi_i(x)}{u_i(x,t)} = \frac{\partial\psi_i(x_0)/\partial n}{\partial u_i(x_0,t)/\partial n} \leqslant 1 \tag{4}$$

for sufficiently small $t > 0$. Hence, there exists t_1 such that (2) is well defined and $u_i(x,t) \geqslant \psi_i(x)$ on $\bar{D}_i$ for $0 < t < t_1 \leqslant \bar{t}$ and all $i = 1, \dots, m$.

Differentiating (2), substituting into Eq. (1) and integrating by parts (notice that the boundary values are always zero), we obtain

$$
\begin{aligned}
\frac{d}{dt} g_i^n(t) &= -n \int_{D_i} \frac{\psi_i^{n+2}}{u_i^{n+1}} \bigl(\Delta u_i + f_i(\mathbf{u})\bigr)\,dx \\
&= -n \int_{D_i} \frac{\psi_i^{n+2}}{u_i^{n+1}} f_i(\mathbf{u})\,dx - n(n+1) \int_{D_i} \frac{\psi_i^{n+2}}{u_i^{n+2}} |\nabla u_i|^2\,dx \\
&\quad + n(n+2) \int_{D_i} \frac{\psi_i^{n+1}}{u_i^{n+1}} \nabla u_i \nabla \psi_i\,dx \\
&= -n \int_{D_i} \frac{\psi_i^{n+2}}{u_i^{n+1}} f_i(\mathbf{u})\,dx - n(n+1) \int_{D_i} \frac{\psi_i^{n}}{u_i^{n+2}} |\psi_i \nabla u_i - u_i \nabla \psi_i|^2\,dx \\
&\quad + n(n+1) \int_{D_i} \frac{\psi_i^{n}}{u_i^{n}} |\nabla \psi_i|^2\,dx - n^2 \int_{D_i} \frac{\psi_i^{n+1}}{u_i^{n+1}} \nabla u_i \nabla \psi_i\,dx \\
&= -n \int_{D_i} \frac{\psi_i^{n+2}}{u_i^{n+1}} f_i(\mathbf{u})\,dx - n(n+1) \int_{D_i} \frac{\psi_i^{n}}{u_i^{n+2}} |\psi_i \nabla u_i - u_i \nabla \psi_i|^2\,dx \\
&\quad - n \int_{D_i} \frac{\psi_i^{n+1}}{u_i^{n}} \Delta \psi_i\,dx \\
&\leqslant -n \int_{D_i} \frac{\psi_i^{n+2}}{u_i^{n}} \left[\frac{f_i(\mathbf{u})}{u_i} - \frac{f_i(\boldsymbol{\psi})}{\psi_i} \right] dx \leqslant 0
\end{aligned} \tag{5}
$$

for any $0 < t \leqslant t_1$. Thus we get

$$g_i^n(t) \leqslant g_i^n(0)$$

for $t \in (0, t_1]$. Taking the nth roots and letting $n \to \infty$, we have

$$\frac{\psi_i(x)}{u_i(x,t)} \leqslant \max_{\bar{D}_i} \frac{\psi_i(x)}{u_i(x,t)} \leqslant \max_{\bar{D}_i} \frac{\psi_i(x)}{\phi_i(x)} \leqslant \frac{1}{1+\varepsilon_0} \tag{6}$$

for $t \in [0, t_1]$. By the continuity of u_i and $\partial u_i / \partial n$, there exists $t_2 > t_1$, such that $u_i(x,t) \geqslant \psi_i(x)$ for $t_1 < t \leqslant t_2$ and $i = 1, \dots, m$. Therefore, (5) and (6) hold for $0 < t \leqslant t_2$. According to this method we can extend u_i to $(0, T^*)$ and (6) holds for all $0 < t < T^*$. We need to show that T^* is a finite number. From (6) and assumptions (i) and (iii), we have

$$1 - \frac{f_i(\boldsymbol{\psi}(x))/\psi_i(x)}{f_i(\mathbf{u}(x,t))/u_i(x,t)} \begin{cases} > 0 & \text{in } D_i, \\ \geqslant c_2 > 0 & \text{on } \bar{B}_r. \end{cases}$$

Note that $u_i > \psi_i$ on $D_j - D_i$ if $D_j - D_i \neq \emptyset$ and $i \neq j$ and

$$u_i(x,t) \geqslant \psi_i(x) \geqslant c_3 > 0 \quad \text{in } \bar{B}_r \text{ for } i = 1, \ldots, m.$$

Choosing $n = \sigma - 1$ in (5), we obtain, by assumption (ii),

$$\begin{aligned}\frac{d}{dt} g_k^{\sigma-1}(t) &\leqslant -(\sigma-1) \int\limits_{D_k} \psi_k^{\sigma+1} \frac{f_k(\mathbf{u})}{u_k^\sigma} \left(1 - \frac{f_k(\boldsymbol{\psi})/\psi_k}{f_k(\mathbf{u})/u_k}\right) dx \\ &\leqslant -(\sigma-1) \int\limits_{B_r} \psi_k^{\sigma+1} \frac{f_k(\mathbf{u})}{u_k^\sigma} \left(1 - \frac{f_k(\boldsymbol{\psi})/\psi_k}{f_k(\mathbf{u})/u_k}\right) dx \leqslant -c \int\limits_{B_r} \psi_k^{1+\sigma}\, dx.\end{aligned}$$

Hence,

$$0 < g_k^{\sigma-1}(t) \leqslant g_k^{\sigma-1}(0) - ct \int\limits_{B_r} \psi_k^{1+\sigma}\, dx \quad \text{or} \quad t \leqslant \frac{g_k^{\sigma-1}(0)}{c \int_{B_r} \psi_k^{1+\sigma}\, dx},$$

which means that T^* cannot increase to infinity. Therefore, $u_k(x,t)$ must blow up in finite time. □

Remark. At first glance, assumption (iii) may seem strong and complicated. However, it is reasonable and almost sharp (see Example 1 below), though lacking theoretic support. In fact, for a complicated domain Ω, it is not clear whether a positive solution of $\Delta\psi_i + f_i(\boldsymbol{\psi}) \geqslant 0$ in Ω, with $\psi_i|_{\partial\Omega} = 0$ for $i = 1, \ldots, m$, exists, or whether such solution, if it does exist, is unique. However, we can easily find a positive solution of $\Delta\psi_i + f_i(\boldsymbol{\psi}) \geqslant 0$ in D_i, with $\psi_i|_{\partial D_i} = 0$ for $i = 1, \ldots, m$, where D_i is any subdomain of Ω, especially, when D_i is a ball. It is clear that the solution ψ_i is negative outside D_i but not too far (see the dotted line in Figs. 1 and 2). So the condition $\psi_i \leqslant 0$ on $D_j - D_i$ if $D_j - D_i \neq \emptyset$ and $i \neq j$ is automatically satisfied.

Example 1. In the radially symmetric case, we can choose $\Omega = \{x \in R^3 \mid |x| < 10\}$, $m = 2$, $f_1(u_1, u_2) = u_1^3(1 + u_2)$ and $f_2(u_1, u_2) = u_1 u_2$. The functions $\psi_1(x)$ and $\psi_2(x)$ are obtained by solving the following ODE equation:

$$\Delta\psi_i(x) + f_i(\psi_1, \psi_2) = \psi_i''(r) + \frac{2}{r}\psi_i' + f_i(\psi_1, \psi_2) = 0, \tag{7}$$

with the initial values $\phi_1(0) = 1.3$, $\psi_2(0) = 0.8$ and $\psi_i'(0) = 0$ for $i = 1, 2$. Then $\psi_1(r)$ is positive for $r < 4.4550$ and negative for $4.4550 < r < 10$; $\psi_2(r)$ is positive for $r < 5.3066$ and negative for $5.3066 < r < 10$. Let $D_1 = \{x \in R^3 \mid |x| < 4.4550\}$ and $D_2 = \{x \in R^3 \mid |x| < 5.3066\}$. The initial conditions $\phi_i(r)$ are constructed from $\psi_i(r)$ as follows:

$$\phi_1(r) = \begin{cases} \psi_1(r) + 0.01 & \text{if } 0 \leqslant r \leqslant 4, \\ 6.1682 - \sqrt{6.1463^2 - (r - 4.5598)^2} & \text{if } 4 < r \leqslant 4.5352, \\ 0.004(10 - r) & \text{if } 4.5352 < r \leqslant 10, \end{cases}$$

$$\phi_2(r) = \begin{cases} \psi_2(r) + 0.01 & \text{if } 0 \leqslant r \leqslant 5, \\ 5.4044 - \sqrt{5.3869^2 - (r - 5.3378)^2} & \text{if } 5 < r \leqslant 5.3378, \\ 0.00375(10 - r) & \text{if } 5.3378 < r \leqslant 10, \end{cases}$$

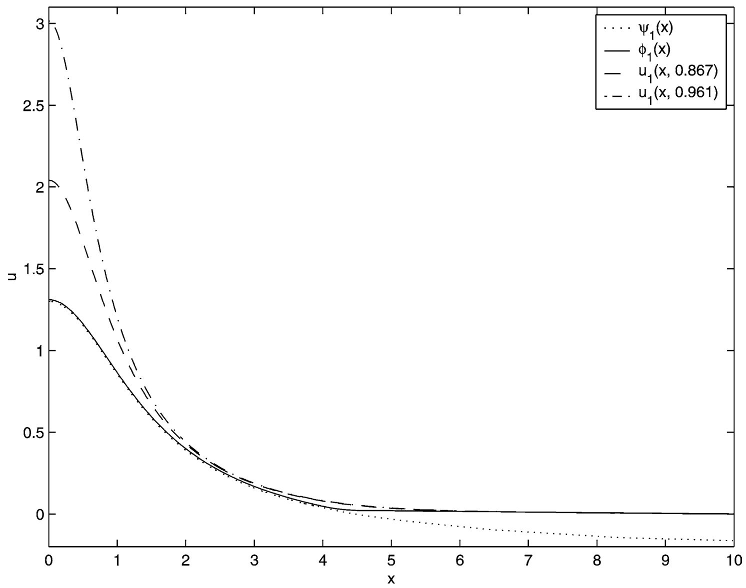

Fig. 1. The solution u_1 moves up as t increases.

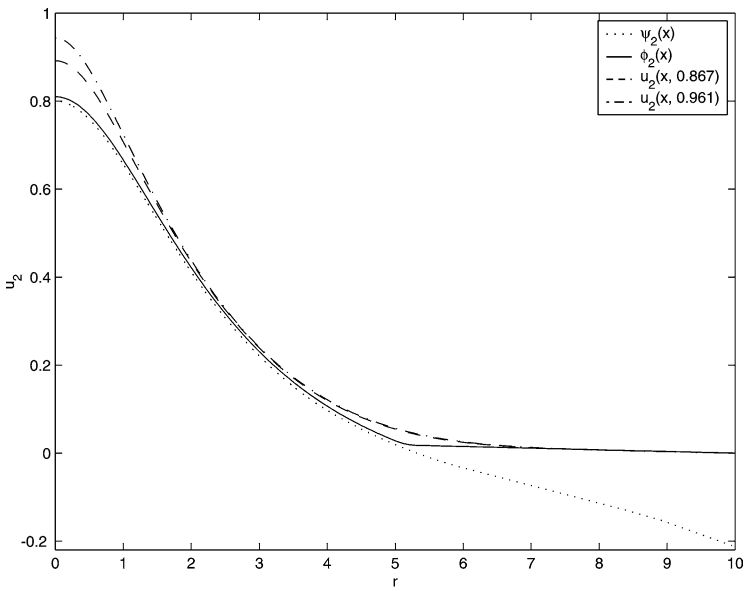

Fig. 2. The solution u_2 moves up as t increases.

so that $\phi_i(x) \in C_0^1(\bar{\Omega})$ and $\phi_i(r) > 1.005\psi_i(r)$ in D_i, $i = 1, 2$. The numerical computations show that the solution of (1) blows up near $T^* = 1.01153$. In Fig. 1, the dotted curve, the solid curve, the dashdot curve and the dashed curve represent $\psi_1(x)$, $\phi_1(x)$,

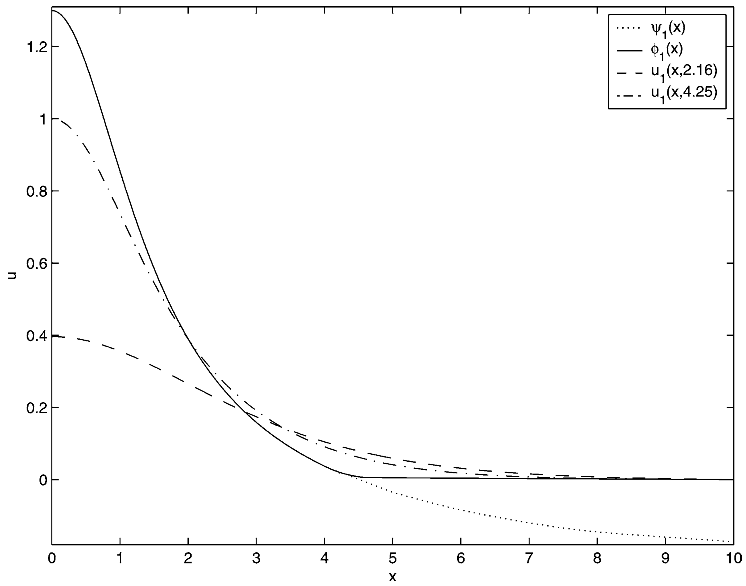

Fig. 3. The peak of the solution u_1 moves down as t increases.

$u_1(x, 0.867)$, and $u_1(x, 0.961)$, respectively. In Fig. 2, similar representations are used for $\psi_2(x)$, $\phi_2(x)$, $u_2(x, 0.867)$, and $u_2(x, 0.961)$. However, the numerical computations show that, if $\phi_i(x) = \psi_i(x) - 0.001$ inside D_i and $\phi_i(x) > 0$ in Ω, then the solutions are global and approach 0 as the time $t \to \infty$ (see Fig. 3).

If $m = 1$, i.e., one equation in (1), then we can allow the initial value $\phi(x)$ to be negative in some subset of Ω. We assume that

(i′) f is differentiable and $f(u)/u > f(v)/v$ if $u > v > 0$.
(ii′) $f(u)/u^\sigma \geqslant c_0 > 0$ for some $\sigma > 1$ and $u > 0$.
(iii′) $D \subset \Omega$ is an open domain in R^n with a closed, simple and smooth boundary ($\partial D \cap \partial\Omega$ may be nonempty). ψ is a solution of $\Delta\psi + f(\psi) \geqslant 0$ in Ω, positive in D and $\psi|_{\partial D} = 0$.
(iv′) $\phi(x) \in C^{1+\alpha}(\bar{\Omega})$ with $\phi|_{\partial\Omega} = 0$ and $\alpha > 0$.

Theorem 2. *Under the hypotheses* (i′)–(iv′), *if* $\phi(x) \geqslant (1 + \varepsilon_0)\psi(x)$ *in* Ω *for some small* $\varepsilon_0 > 0$, *then the solution* u *of* (1) *with* $m = 1$ *must blow up in finite time.*

Proof. Let $v(x,t) = u(x,t) - \psi(x)$. Then

$$v_t - \Delta v = u_t - \Delta u + \Delta\psi \geqslant f(u) - f(\psi) = \int_0^1 f'\big(\psi + \xi(u - \psi)\big)\,d\xi \times v(x,t).$$

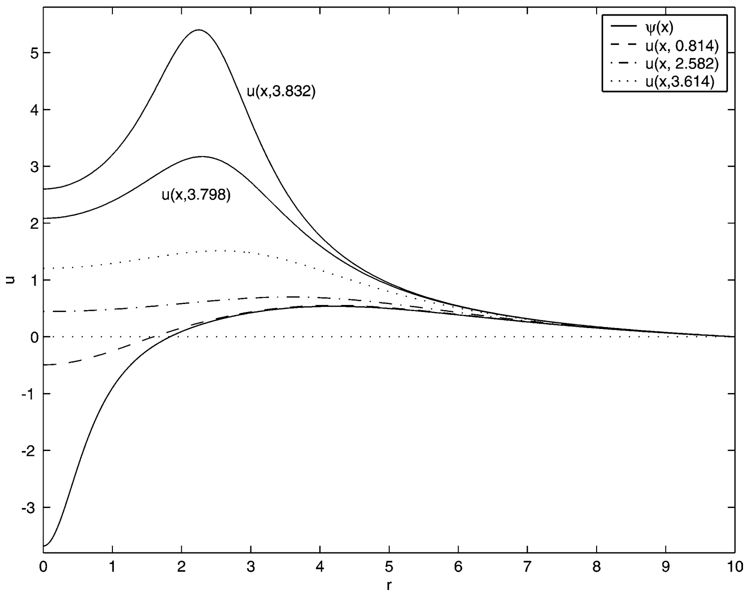

Fig. 4. The blowup point forms as t increases.

By the maximum principle, $v(x,t) > 0$ or $u(x,t) > \psi(x)$ in $\Omega \times (0, T^*)$. Let

$$g_n(t) = \int_D \frac{\psi^{n+2}(x)}{u^n(x,t)}\,dx.$$

Then $g_n(t)$ is well defined in $(0, T^*)$ because $u(x,t) > \psi(x) > 0$ in D. The rest of the proof is similar to that of Theorem 1. $\square$

Example 2. If $m = 1$ and $f(u) = u^3$, we can choose $\Omega = \{x \in R^3 \mid |x| < 10\}$ and $\psi(r)$ to be the solution of

$$\psi''(r) + \frac{2}{r}\psi' + \psi^3 = 0$$

with the initial values $\psi(0) = -3.7$ and $\psi'(0) = 0$ (see Fig. 4). Then we consider the problem

$$\begin{aligned} &u_t = u''(r) + \frac{2}{r}u' + u^3,\\ &u(10,t) = 0, \quad u'(0,t) = 0,\\ &u(r,0) = \phi(r), \end{aligned} \tag{8}$$

where $\phi(r) = \psi(r) + 0.0119$ so that $\phi(10) = 0$. To obtain a good numerical solution of (8), we use the moving collocation method developed by Huang and Russell [9], which is based upon the equidistribution principle. The moving collocation method has proved very effective for solving a variety of problems involving blowup solutions of partial

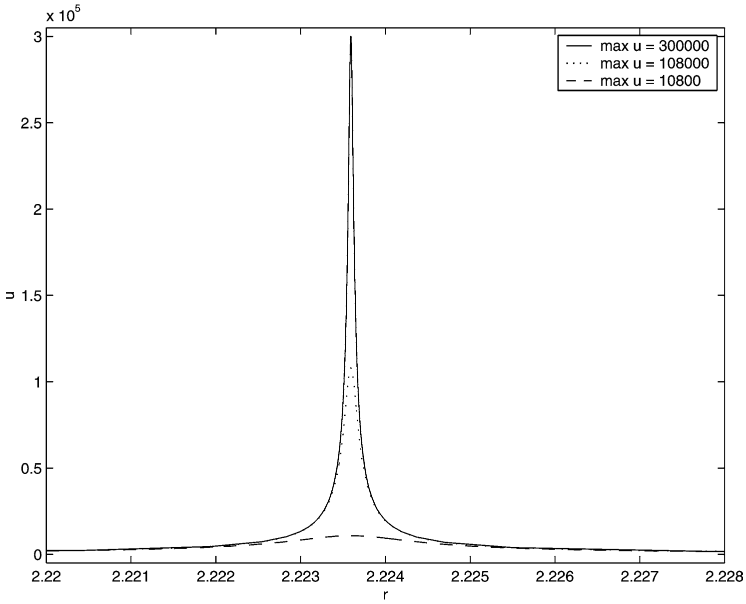

Fig. 5. The singularity is developed as $t \to T^*$.

differential equations. In Fig. 4, the solid curve on the bottom is $\psi(r)$. As t increases, $u(r,t)$ moves up and the blowup point forms near $r_0 = 2.2236$. In Fig. 5, the development of the singularity near r_0 is illustrated when $u(r_0,t) = 10800$, 108000, 300000 and $t = 3.850104652004, 3.850104656298, 3.850104656336$, respectively. Hence, the blowup time is $T^* = 3.8501\ldots$. Since the solution is radially symmetric, the blowup points form a circle $r = r_0$.

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